

Physics

Energy “Bucket” Model

I. Overview

This document provides an overview of the “Energy Bucket” problem solving strategy. This tutorial will help you to solve problems involving the Law of Conservation of Energy.

This guide will introduce the “Energy Bucket Model” problem solving strategy in Part II, provide an example of this problem solving strategy in Part III, and provide a student sample response in Part IV. A list of supplemental problems is included in Part V of the document.

Part II. Energy “Bucket” Model Problem Solving Strategy

A problem solving strategy for addressing problems involving the Law of Conservation of Energy is given below.

Step 1: Draw the reference line (the organizational part)

Problems involving the Law of Conservation of Energy usually involve an object moving from one point (e.g. ground level) to another point above the ground, as in the example a pole vaulter; or an object that is above the ground level which falls toward the ground. To clearly define potential energy values (which depend on the height of the object above a reference), you are required to draw a “Reference Line” on their diagram in the “Given” of the problem. This line is usually the lowest point that the object moves to in the question and is used to clearly define heights, and therefore, potential energy values.

Step 2: Write statement of COE

Here you will state the Law of Conservation of Energy:

$$E_A = E_B$$

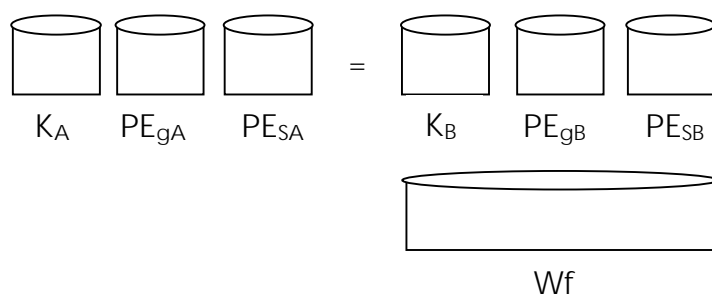
In addition to writing this statement, you will clearly label where "A" and "B" are in the diagram. This might be right before a ball is shot up into the air, the highest point a ball goes when shot up into the air, etc.

Step 3: Draw Buckets (the Physics part)

Problems involving the use of the Conservation of Energy always involve a "beginning point" and an "end point". In the example of a pole vaulter who has kinetic energy just before he leaves the ground and possesses gravitational potential energy at the top of his vault (where he is momentarily at rest), the "beginning point" is the instant before he leaves the ground and the "end point" is the instant when he is at his maximum height and, for a instant, has no velocity or kinetic energy.

The next step of the Energy Bucket model requires students to draw the following set of buckets and identify which forms of energy are present at the "beginning point" or "A" and "end point" or "B" as described above. The definition of each bucket is presented below.

Note that the W_f (work done by friction) "drip pan" is used to "capture" energy that is spilled in "pouring" energy from the PE_g bucket to the K_B bucket (if air resistance or other frictional forces are present), to account for all "energy losses".



The "A" and "B" subscripts represent the "beginning point" and "end point" respectively.

K_A	=	the kinetic energy at point A [J]
PE_{gA}	=	the gravitational potential energy at point A [J]
PE_{sA}	=	the spring potential energy at point A [J]
K_B	=	the kinetic energy at point B [J]
PE_{gB}	=	the gravitational potential energy at point B [J]
PE_{sB}	=	the spring potential energy at point A [J]
W_f	=	the work done by frictional forces [J]

Step 4: Write Energy Equation (the mathematical part)

After filling in the buckets according to which energies are present at the beginning and end points, the corresponding energy equation can be written.

Step 5: Solve (the mathematical part)

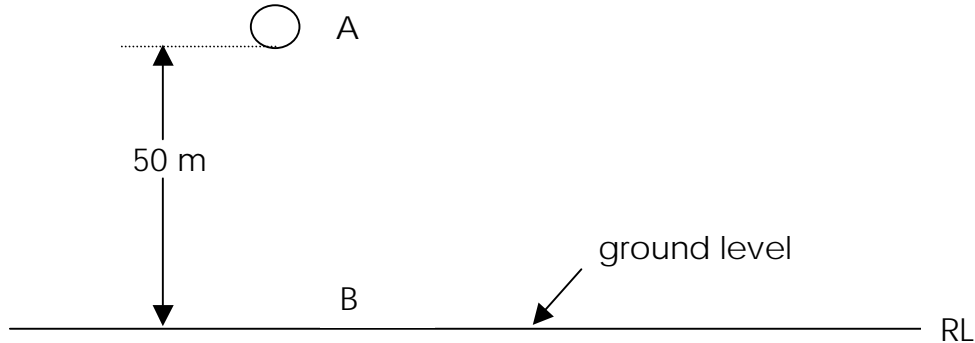
After completing Step 4, the known and unknown variables will be listed. If step 4 is completed correctly, this step concludes with the solving of the algebraic equation for the unknown variable. As mentioned above, this step is a purely mathematical step, using math as a tool to arrive at the correct answer. The following section models how this strategy can be used to solve a Conservation of Energy problem.

Part III. Example Conservation of Energy Problem

An 0.5 kg ball is dropped from rest from a height of 50 m above the ground.

a) What is the velocity of the ball right before it hits the ground assuming no air resistance. b) What is velocity of the ball right before it hits the ground if 100 J of energy are lost due to air resistance?

Given:



Find: a) v_B (no air resistance)
b) v_B (with air resistance)

Solution:

PART A:

1. Draw the reference line

Since the lowest point that the ball reaches in this problem is the ground, the letters "RL" are drawn in the diagram at the ground level in the "Given". This defines where all heights, and therefore, gravitational potential energies will be measured from.

2. Write statement of COE

Here we write:

$$E_A = E_B$$

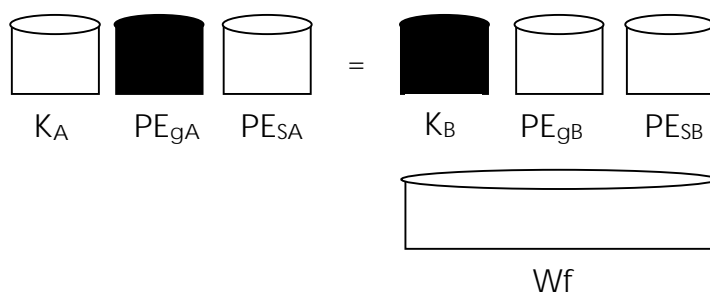
Note that the points "A" and "B" have been identified and labeled in the "Given".

3. Draw Energy buckets

Let's address each of the energy buckets separately:

- K_A = Is the object moving at point A? No, so bucket is not filled in
- PE_{gA} = Is the object above the RL? Yes, so it has PEG, so we fill in the bucket
- PE_{sA} = Is a spring stretched or compressed? No, so bucket is not filled in
- K_B = Is the object moving at point B? Yes, so we fill in the bucket
- PE_{gB} = Is the object above the RL? At point B, it is NOT, so we do not fill in the bucket
- PE_{sB} = Is a string stretched or compressed? No, so bucket is not filled in
- W_f = Is friction or air resistance acting? No, so bucket is not filled in

Therefore, we fill in the buckets as shown below:



It is important to address HOW energy got into the PE_{gA} bucket in the first place. Someone or something had to lift the object above the ground a certain distance. That is, a force was applied to the object over a certain distance, or Work was done ON the object to give it the energy.

4. Write Energy Equation

If the energies at points "A" and "B" are "filled in" correctly, then we can write the resulting Conservation of Energy equation:

$$mgh_A = \frac{1}{2}mv_B^2$$

5. Solve

Again, this step is purely a mathematical step. If the correct buckets have been filled in (conceptual Physics component) and the correct equation has been written, then the solution of v_B is as follows:

$$v_B = \sqrt{2gh_A} = \sqrt{2(9.8)(50)} = 31.3[m / s]$$

PART B:

1. Draw the reference line

Since the ball moves to the same level, the RL does not change. It remains at ground level.

2. Write statement of COE

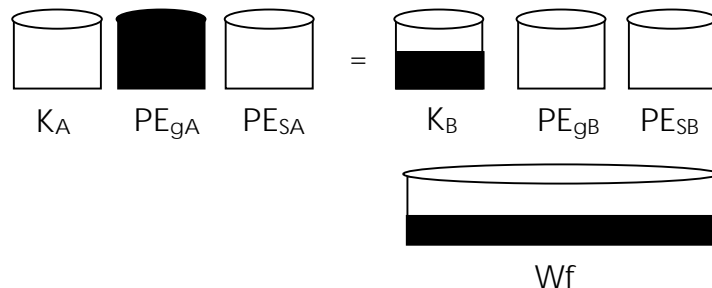
Here we write:

$$E_A = E_B$$

Note that the points "A" and "B" have been identified and labeled in the "Given".

3. Draw Energy buckets

At point "A" we only have U_{gA} , therefore we fill that bucket with energy. At point "B" we have only K_B , therefore we fill that bucket only. However, some energy is "lost" due to work done by air resistance (i.e. some energy is spilled into the "drip pan"), therefore we show that some energy is in the W_f "drip pan". This illustrates that although some energy is "lost", or more correctly, no longer in kinetic or potential form, all of the energy can be accounted for.



4. Write Energy Equation

If the energies at points "A" and "B" are correct, then we write the resulting Conservation of Energy equation:

$$mgh_A = \frac{1}{2}mv_B^2 + W_{air_resistance}$$

5. Solve

Again, this step is purely a mathematical step. If the correct buckets have been filled in (the conceptual Physics component) and the correct equation has been written, then the solution of v_B is as follows:

$$v_B = \sqrt{\frac{2(mgh_A - W_{air})}{m}} = \sqrt{\frac{2((0.5)(9.8)(50) - 100)}{0.5}} = 24.08[m / s]$$

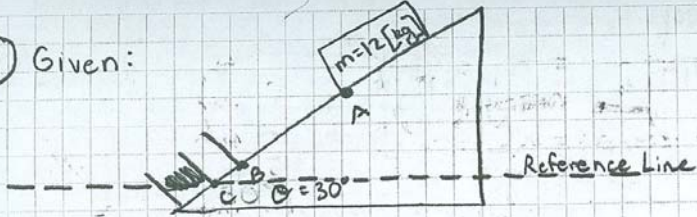
The answer of 24.08 m/s is less than that without air resistance as it should be.

Part IV. Student Sample

The sample problem below illustrates the use of the Energy Bucket Model.

A block of mass $m = 12 \text{ kg}$ is released from rest on a frictionless incline of angle 30 degrees. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N . The block momentarily stops when it compresses the spring by 5.5 cm . (a) How far down does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?

31) Given:



Find: (a) x_{ac}
(b) v_B

Solution: ① Draw Reference Line

② List Equations

$$E_A = E_B$$

$$K_A + U_{gA} + U_{sA} = K_B + U_{gB} + U_{sB}$$

$$mg y_A = \frac{1}{2} m v_B^2 +$$

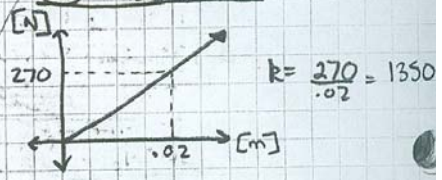
③ List Variables

$$m = 12 \text{ [kg]} \quad k = 1350 \text{ [N/m]}$$

$$g = 9.8 \text{ [m/s}^2\text{]} \quad x_c = .055 \text{ [m]}$$

$$y_A = \text{[] [m]} \quad v_B = \text{[] [m/s]}$$

③B Side Dish



$$k = \frac{270}{.02} = 1350$$

④ Solve

(a) $mg y_A = \frac{1}{2} k x_c^2$

$$y_A = \frac{\frac{1}{2} k x_c^2}{mg} = \frac{\frac{1}{2} \cdot (1350) \cdot (.055)^2}{(12)(9.8)}$$

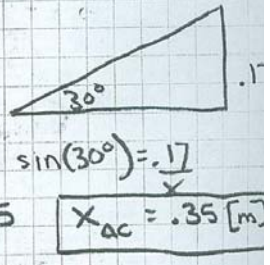
$$y_A = .17 \text{ [m]}$$

(b) $mg y_A = \frac{1}{2} m v_B^2 \rightarrow v_B = \sqrt{\frac{mg y_A \cdot 2}{m}}$

$$v_B = \sqrt{2 \cdot g \cdot y_A} = \sqrt{2 \cdot 9.8 \cdot .17} = 1.83$$

$$v_B = 1.83 \text{ [m/s]}$$

④B Side Dish

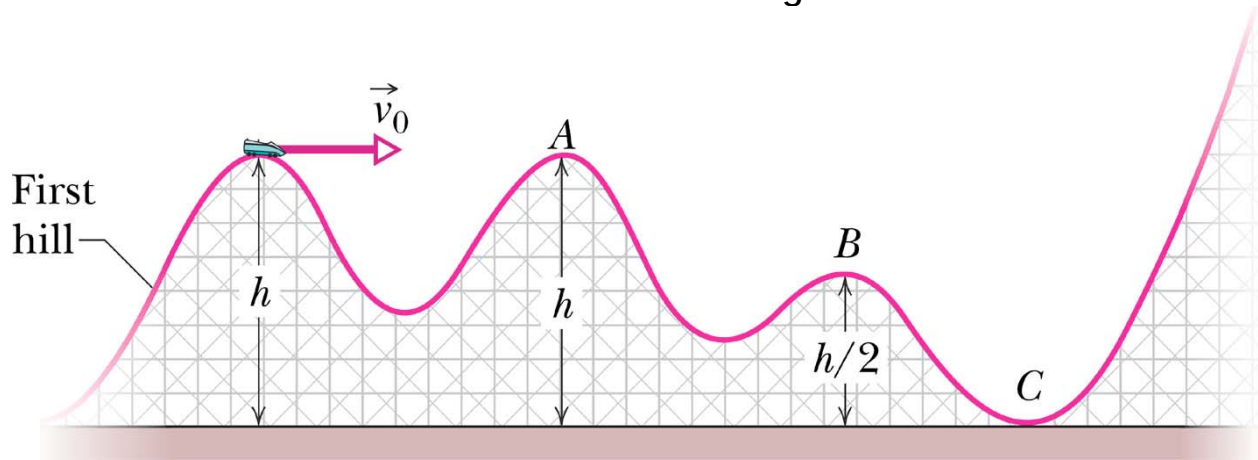


$$\sin(30^\circ) = \frac{.17}{x_{ac}} \rightarrow x_{ac} = .35 \text{ [m]}$$

Part V. Supplemental Problems

This problem set includes a selection of quantitative and qualitative problems involving Work and the Law of Conservation of Energy.

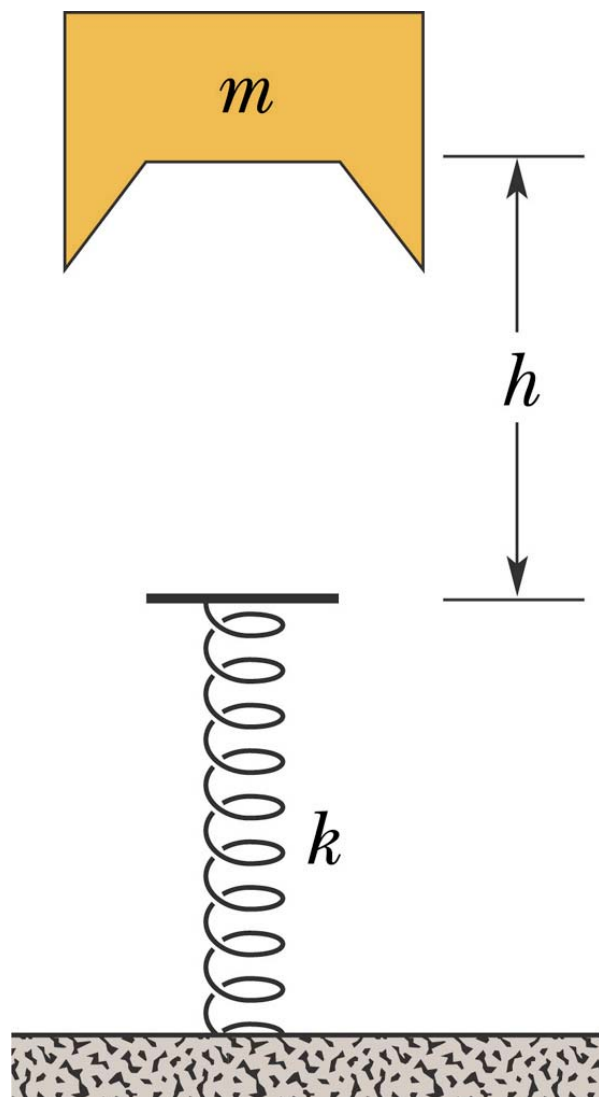
Problems 1 and 2 refer to the figure below



- 1. A frictionless roller-coaster car of mass $m = 825 \text{ kg}$ tops the first hill with a speed $v_0 = 17.0 \text{ m/s}$ at height $h = 42 \text{ m}$. What will be the car's speed at point B?**
 - a. 12.34 [m/s]
 - b. 20.28 [m/s]
 - c. 26.46 [m/s]
 - d. 33.21 [m/s]
 - e. 41.60 [m/s]

- 2. What will the car's speed be at point C?**
 - a. 12.34 [m/s]
 - b. 20.28 [m/s]
 - c. 28.69 [m/s]
 - d. 33.35 [m/s]
 - e. 41.60 [m/s]

3. A block of mass $m = 2.0$ kg is dropped from height $h = 40$ cm onto a spring of spring constant $k = 1960$ N/m. Find the maximum distance the spring is compressed.



- a. 2.2 [m]
- b. 1.4 [m]
- c. 1.0 [m]
- d. 0.7 [m]
- e. 0.1 [m]

Solution for #2 above (involves quadratic formula)

22. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$
$$0 = -mg(h + x) + \frac{1}{2}kx^2$$

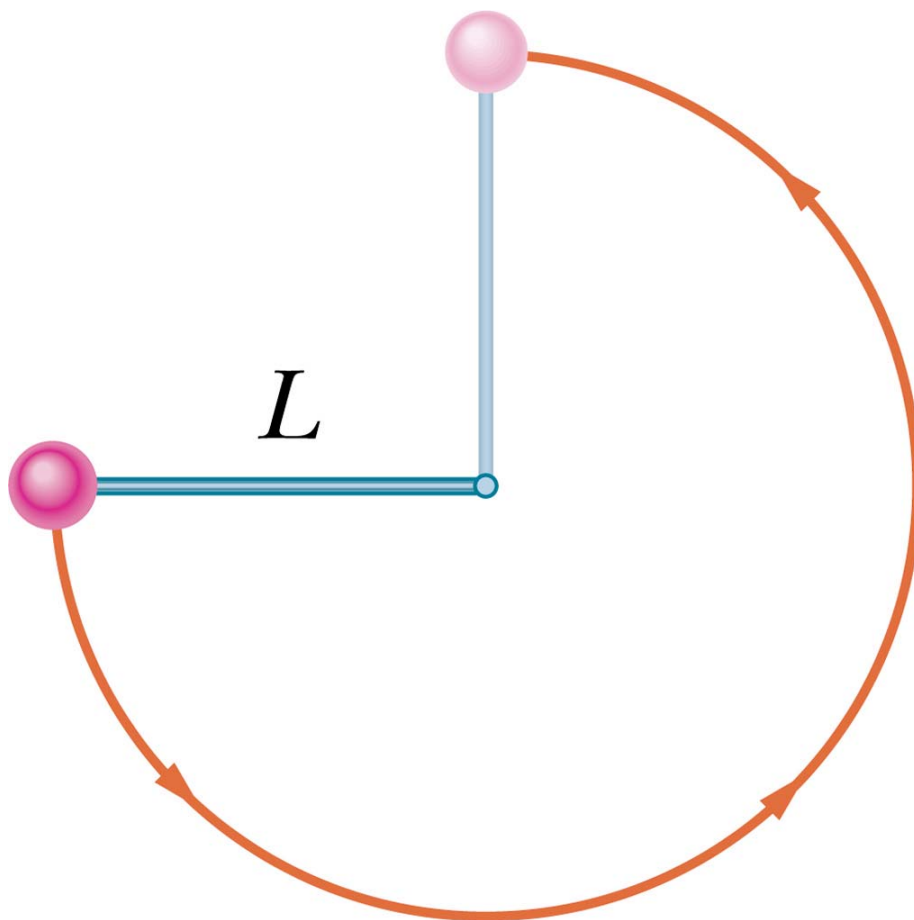
which is a second degree equation in x . Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

Now $mg = 19.6$ N, $h = 0.40$ m, and $k = 1960$ N/m, and we choose the positive root so that $x > 0$.

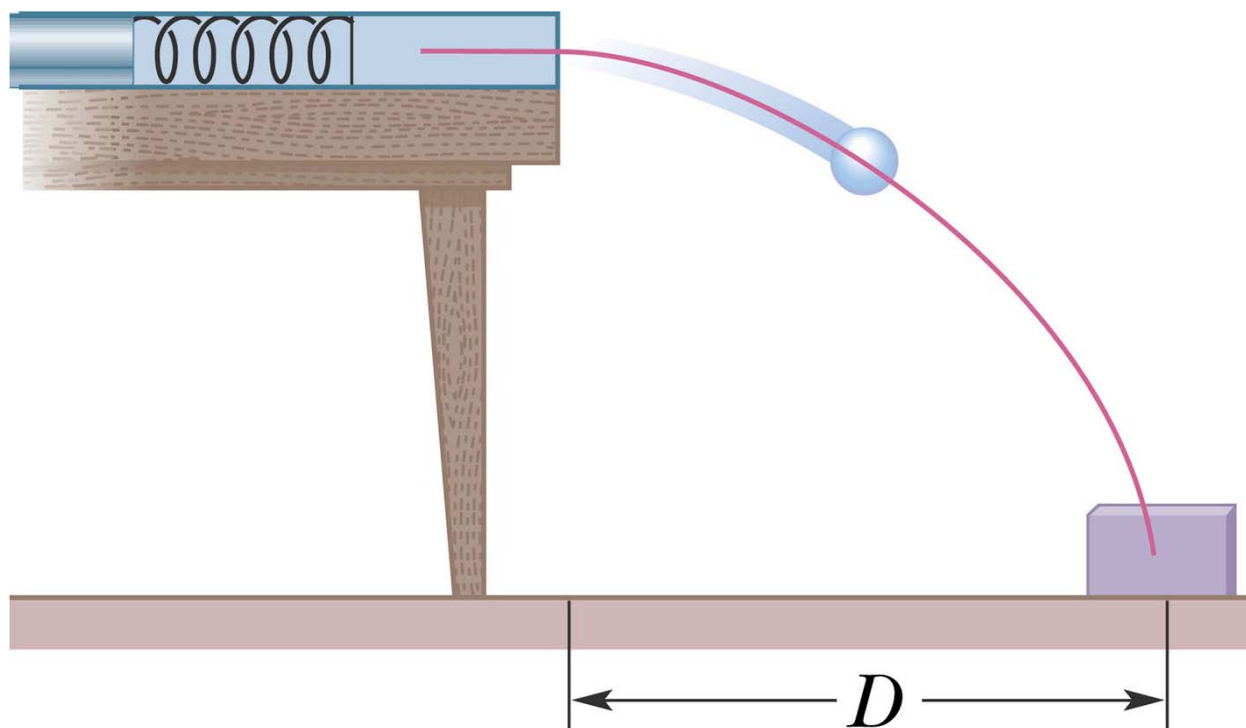
$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10 \text{ m}$$

4. The figure below shows a ball with mass $m = 3.0 \text{ kg}$ attached to the end of a thin rod with length $L = 2.5 \text{ m}$ and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held horizontally as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertical position, with zero speed there. What velocity did the push give the ball just as it left the horizontal position?



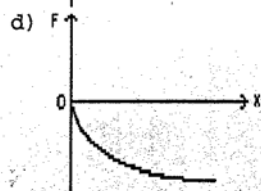
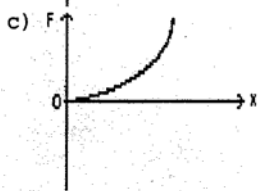
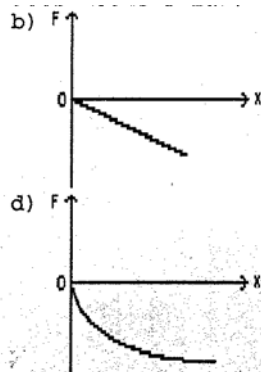
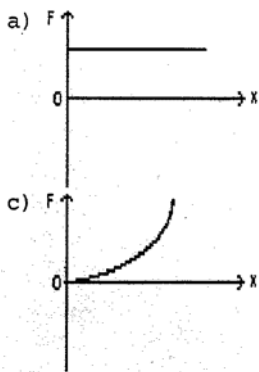
- a. 4.94 [m/s]
- b. 6.12 [m/s]
- c. 7.00 [m/s]
- d. 37.45 [m/s]
- e. 49.00 [m/s]

Problems 5 and 6 refer to the diagram below



5. The spring of a projectile launcher ($k = 500 \text{ N/m}$) is stretched back 0.25 m and shoots a ball of mass $m = 3.4 \text{ kg}$ off of a table. The launcher is 1.5 above the ground level. What is the velocity of the ball just as it leaves the table?
- 10.81 [m/s]
 - 9.19 [m/s]
 - 3.03 [m/s]
 - 1.50 [m/s]
 - 0.71 [m/s]
6. The ball hits a box a distance D from the base of the table. What is the distance D ?
- 0.69 [m]
 - 0.83 [m]
 - 1.01 [m]
 - 1.68 [m]
 - 3.03 [m]

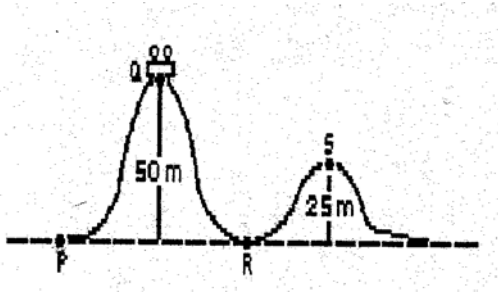
7. Two mice, Pinky and Brain, push against a stationary wall, Pinky stops after 10 minutes, while Brain is able to push for 5 minutes longer. Compare the work they do.
- Pinky does 50% more work than Brain
 - Brain does 50% more work than Pinky
 - Pinky does 75% more work than Brain
 - Neither of them do any work
8. You lift a 10 N physics book up in the air a distance of 1m, at a constant velocity of 0.5 m/s. The work done by gravity is:
- +10 J
 - 10 J
 - +5 J
 - 5 J
9. The area under a curve, on a Force-position (F vs. x graph) represents:
- work
 - power
 - kinetic energy
 - efficiency
10. Which of the following curves illustrates Hooke's Law?



11. A 10 kg mass, hung onto a spring, causes the spring to stretch 2 cm. The spring constant is:
- 49 N/cm
 - 5 N/cm
 - 0.2 N/cm
 - 0.02 N/cm

12. Car A moves twice as fast as car B, and car A has half the mass of car B. The kinetic energy of car A, compared to car B is:
- the same
 - 2 to 1
 - 4 to 1
 - 1 to 2
13. An acorn falls from a tree. Compare its kinetic energy K , to its potential energy PE due to gravity.
- K increases, and PE_{gravity} decreases
 - K decreases, and PE_{gravity} decreases
 - K increases, and PE_{gravity} increases
 - K decrease, and PE_{gravity} increases
14. Matthew pulls his little sister Sarah in a sled on an icy surface (assume no friction), with a force of 60 N at an angle of 37° upward from the horizontal. If he pulls her a distance of 12 m, the work he does is:
- 185 J
 - 433 J
 - 575 J
 - 720 J
15. A 4.0 kg box of fruit slides 8.0 m down a ramp, inclined at 30° from the horizontal. If the box slides down at a constant velocity of 5 m/s (with friction present), the work done by gravity is:
- zero
 - +78 [J]
 - 78 [J]
 - +157 [J]
 - 157 [J]
16. A 200 g mass attached to the end of a spring causes it to stretch 5.0 cm. If another 200 g mass is added to the spring, the potential energy of the spring will be:
- the same
 - twice as much
 - 3 times as much
 - 4 times as much

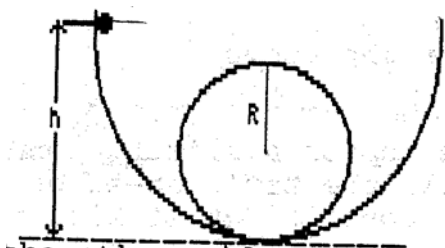
17. You throw a ball straight up. Compare the sign of the work done by gravity while the ball goes up with the sign of the work done by gravity while it goes down.
- a. Work up is +, and the work down is +
 - b. Work up is +, and the work down is -
 - c. Work up is -, and the work down is +
 - d. Work up is -, and the work down is -
18. How much work is done by gravity on the 1000 kg roller coaster as it moves from point Q to point R?



- a. 32,000 [J]
 - b. 50,000 [J]
 - c. 245,000 [J]
 - d. 490,000 [J]
 - e. 980,000 [J]
19. If the roller coaster leaves point Q from rest, how fast is it traveling at point R? Ignore friction.
- a. 13.4 [[m/s]
 - b. 22.1 [m/s]
 - c. 31.3 [m/s]
 - d. 490 [m/s]
 - e. 980 [m/s]
20. If the roller coaster leaves point Q from rest, what is its speed at point S (at the top of the 25 m peak)?
- a. 13.4 [m/s]
 - b. 22.1 [m/s]
 - c. 31.3 [m/s]
 - d. 490 [m/s]
 - e. 980 [m/s]

21. A container of water is lifted vertically 3 m, then returned to its original position. If the total weight is 30 N, how much total work was done by gravity?
- a. 45 [J]
 - b. 90 [J]
 - c. 0 [J]
 - d. -45 [J]
 - e. -90 [J]
22. On a plot of F vs. x , what represents the work done by the force F ?
- a. The slope of the curve
 - b. The length of the curve
 - c. The area under the curve
 - d. The product of the maximum force times the maximum x
23. A 1 kg flashlight falls to the floor. At the point during its fall, when it is 0.70 m above the floor, its potential energy exactly equals its kinetic energy. How fast is it moving?
- a. 3.7 [m/s]
 - b. 6.9 [m/s]
 - c. 13.7 [m/s]
 - d. 44.8 [m/s]
 - e. 56.9 [m/s]
24. A truck weighs twice as much as a car, and is moving at twice the speed of the car. Which statement is true about the truck's kinetic energy (K) compared to that of the car?
- a. All that can be said is that the truck has more K
 - b. The truck has twice the K of the car
 - c. The truck has 4 times the K of the car
 - d. The truck has 8 times the K of the car

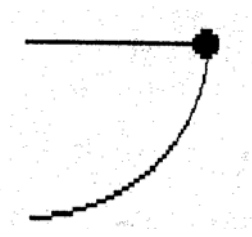
25. A ball is released from rest at the left side of the loop-the-loop, at the height shown. If the radius of the loop is R , what vertical height does the ball rise to on the other side neglecting friction?



Note: h is much higher than R

- a. Less than R
b. R
c. $2R$
d. Greater than $2R$
26. A spring is characterized by a spring constant of 60 N/m . How much potential energy does it store, when stretched by 1 cm ?
- a. 0.003 [J]
b. 0.3 [J]
c. 60 [J]
d. 600 [J]
e. 6000 [J]
27. You and your friend want to go to the top of the Eiffel Tower. Your friend takes the elevator straight up. You decide to walk up the spiral stairway, taking longer to do so. Compare the gravitational potential energy of you and your friend, after you both reach the top. Assume you and your friend weigh the same.
- a. Your friend's PEG is greater than your PEG, because she got to the top faster.
b. Your PEG is greater than your friend's PEG because you traveled a greater distance getting to the top.
c. Both of you have the same amount of potential energy
d. It is impossible to tell, since the time and distances are unknown
28. A skier of mass 40 kg pushes off the top of a hill with an initial speed of 4 m/s . Neglecting friction, how fast will she be moving after dropping 10 m in elevation?
- a. 7.11 [m/s]
b. 10.29 [m/s]
c. 14.56 [m/s]
d. 49.91 [m/s]
e. 196.12 [m/s]

29. A simple pendulum, consisting of a mass m , is attached to the end of a 1.5 m length of string. If the mass is held out horizontally, and then released from rest, its speed at the bottom is:



- a. 5.4 [m/s]
b. 9.8 [m/s]
c. 17 [m/s]
d. 96 [m/s]
30. A toy rocket, weighing 10 N, blasts straight up from ground level with a kinetic energy of 40 J. At the exact top of its trajectory, its total mechanical energy is 140 J. To what vertical height does it rise?
a. 10 [m]
b. 14 [m]
c. 24 [m]
d. None of the above
31. King Kong falls from the top of the Empire State Building through the air to the ground below (air friction is present). How does his kinetic energy (K) just before striking the ground compare to his potential energy (PE) at the top of the building?
a. K is equal to PE
b. K is greater than PE
c. K is less than PE
d. It is impossible to tell without knowing the mass of King Kong and the height of the building
32. A 30 N stone is dropped from a height of 10 m, and strikes the ground with a velocity of 7 m/s. What average force of air friction acts on it as it falls?
a. 22.5 N
b. 75 N
c. 225 N
d. 293 N

33. The speed needed at the bottom of a loop-the-loop track so that a car can coast to the top, with sufficient speed to stay on the track, depends on the mass of the car. (Neglect the effects of friction and air resistance)
- Always true
 - Never true
 - Sometimes true, since kinetic energy is a function of mass
 - Sometimes true, since potential energy is a function of mass
34. An object is lifted vertically 2 m and held there. If the object weighs 120 N, how much work was done in lifting it?
- 2352 [J]
 - 240 [J]
 - 0 [J]
 - 280 [J]
 - 2352 [J]
35. A projectile of mass m leaves the ground with a kinetic energy of 220 J. At the highest point in its trajectory, its kinetic energy is 120 J. To what vertical height, relative to its launch point, did it rise?
- $220/mg$ [m]
 - $120/mg$ [m]
 - $100/mg$ [m]
 - $50/mg$ [m]
 - $25/mg$ [m]
36. Which of the following is not a good example of potential energy?
- A man on the ledge of a building
 - A stretched spring
 - A car moving on a flat road
 - Water at the top of a waterfall
37. Describe the energy of a car driving up a hill.
- entirely kinetic
 - entirely potential
 - both kinetic and potential
 - gravitational

Energy Model Answer Key

1	C
2	D
3	E
4	C
5	C
6	D
7	D
8	B
9	A
10	B
11	A
12	B
13	A
14	C
15	D
16	D
17	C
18	D
19	C
20	B
21	C
22	C
23	A
24	D
25	D
26	A
27	C
28	C
29	A
30	D
31	C
32	A
33	B
34	B
35	C
36	C
37	C