

THE WORK INTEGRAL

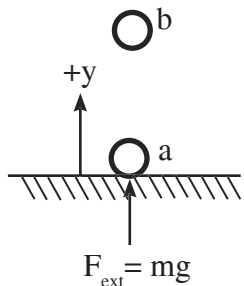
CASE 1: CONSTANT GRAVITATIONAL FORCE

$$W_{F_{ext}} = \int_a^b F(y) \cdot dy$$

$$W_{F_{ext}} = \int_a^b |mg| |dy| \cos 0$$

$$W_{F_{ext}} = mgy \Big|_a^b = mgb - mga$$

$$W_{F_{ext}} = \Delta U = "+ "$$



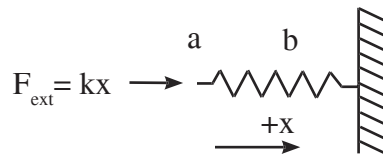
CASE 2: SPRING FORCE

$$W_{F_{ext}} = \int_a^b F(x) \cdot dx$$

$$W_{F_{ext}} = \int_a^b |kx| |dx| \cos 0$$

$$W_{F_{ext}} = \frac{1}{2} kx^2 \Big|_a^b = \frac{1}{2} kb^2 - \frac{1}{2} ka^2$$

$$W_{F_{ext}} = \Delta U = "+ "$$



CASE 3: NON-CONSTANT GRAVITATIONAL FORCE

$$W_{F_{ext}} = \int_a^b F(r) \cdot dr$$

$$W_{F_{ext}} = \int_a^b \left| \frac{GMm}{r^2} \right| |dr| \cos 0$$

$$W_{F_{ext}} = -\frac{GMm}{r} \Big|_a^b = \left(-\frac{GMm}{b} \right) - \left(-\frac{GMm}{a} \right)$$

$$W_{F_{ext}} = \Delta U = "+ "$$

